Microscopic Pictures of Dynamical Symmetry Breaking in Supersymmetric $SU(n_c)$, $USp(2n_c)$ and $SO(n_c)$ Theories

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ABSTRACT: Several distinct mechanisms of confinement and dynamical symmetry breaking (DSB) are identified, in a class of supersymmetric $SU(n_c)$, $USp(2n_c)$ and $SO(n_c)$ gauge theories. In some of the vacua, the magnetic monopoles carrying nontrivial flavor quantum numbers condense, causing confinement and symmetry breaking simultaneously. In more general classes of vacua, however, the effective low-energy degrees of freedom are found to be constituents of the monopoles - dual (magnetic) quarks. These magnetic quarks condense and give rise to confinement and DSB. We find two more important classes of vacua, one is in various universality classes of nontrivial superconformal theories (SCFT), another in free-magnetic phase.

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1 Introduction

The aim of our study here is to continue the search for better understanding on the physics in the infrared of non-Abelian asymptotic free gauge theories (e.g., QCD). The questions of central interests are: i) the mechanism of confinement; ii) the mechanism of flavor (chiral) symmetry breaking; and the relation between the two; iii) the existence of other phases (CFT, oblique confinement, etc.), and iv) the θ dependence, CP properties, etc. There has been substantial progress recently in this field, coming from the study of supersymmetric models.[1, 2, 3, 4, 5, 6] For instance, an interesting hint came from the study of $N = 2$ supersymmetric QCD with $SU(2)$ gauge group[1, 2], in which supersymmetry is softly broken to $N = 1$: in some of the vacua, condensation of magnetic monopoles leads to confinement and flavor symmetry breaking simultaneously.

We study here more general classes of $N = 2$ supersymmetric $SU(n_c)$, $USp(2n_c)$ and $SO(n_c)$ gauge theories with n_f quarks[7, 8], with a small adjoint mass breaking supersymmetry to $N = 1$. The generalizations turn out to be highly nontrivial, and the resulting variety of dynamical possibilities much richer than might be expected from the $SU(2)$ cases studied by Seiberg and Witten[1, 2], or from the pure ($n_f = 0$) $SU(n_c)$ theory discussed by Douglas and Shenker[3]. For the exact Seiberg-Witten curves for $N = 2$ supersymmetric $SU(n_c)$, $USp(2n_c)$ and $SO(n_c)$ gauge groups see [4].

Before presenting the model and discuss our main results, let us mention a recent work in which the interrelation between confinement and chiral symmetry breaking in the standard (non-supersymmetric) QCD with $SU(2)$ gauge group, was studied [9], by using the Faddeev-Niemi gauge field decomposition. It is argued that in the ground state of ($SU(2)$) QCD there are two complementary, competing configurations which are important: one - meronlike configurations or regularized Wu-Yang monopoles - responsible for confinement but in itself has nothing to do with chiral symmetry breaking, while the instantonlike configurations are fundamental for the chiral symmetry breaking but are themselves unrelated to confinement.

1.1 The model

The models discussed here are described by the Lagrangian,

$$L = L^{(N=2)}(W, \Phi, \tilde{Q}_i, Q^i) + m_i \tilde{Q}_i Q^i|_F + \mu \Phi^2|_F, \tag{1.1}$$

with $m_i, \mu \ll \Lambda$, where the first term of the standard $N = 2$ supersymmetric Lagrangian with massless hypermultiplets (quarks) in the fundamental representation of the gauge group, m_i is the bare quark mass of the i-th flavor, and the adjoint mass μ breaks supersymmetry to $N = 1$. The models have no flat directions so that there are finite number of isolated $N = 1$ vacua, keeping track of which provides us with a quite nontrivial check of our analyses. Also, only those theories are considered in which the interactions become strong in the infrared. The global symmetry of the models are:

$$\begin{aligned} SU(n_c): \qquad G_F &= U(n_f) \quad (m_i \to m, \text{ or } 0); \\ USp(2n_c): \qquad G_F &= SO(2n_f) \quad (m_i \to 0); \end{aligned}$$

$$SO(n_c): \qquad G_F \;=\; USp(2n_f) \quad (m_i \to 0). \tag{1.2}$$

Also, the global discrete symmetries such as $Z_{2n_c-n_f}$ in $SU(n_c)$ play important roles.

1.2 The results

The most striking results of our analysis, summarized in Table 1 and Table 2 for $SU(n_c)$ and $USp(2n_c)$ theories, are the following.

Deg.Freed.	Eff. Gauge Group	Phase	Global Symmetry
monopoles	$U(1)^{n_c-1}$	Confinement	$U(n_f)$
monopoles	$U(1)^{n_c-1}$	Confinement	$U(n_f-1)\times U(1)$
dual quarks	$SU(r)\times U(1)^{n_c-r}$	Confinement	$U(n_f-r)\times U(r)$
rel. nonloc.	-	Almost SCFT	$U(n_f/2)\times U(n_f/2)$
dual quarks	$SU(\tilde{n}_c)\times U(1)^{n_c-\tilde{n}_c}$	Free Magnetic	$U(n_f)$

Table 1: Phases of $SU(n_c)$ gauge theory with n_f flavors. The label r in the third row runs for $r = 2, 3, \ldots, [\frac{n_f-1}{2}]$. "rel. nonloc." means that relatively nonlocal monopoles and dyons coexist as low-energy effective degrees of freedom. "Confinement" and "Free Magnetic" refer to phases with $\mu \neq 0$. "Almost SCFT" means that the theory is a non-trivial superconformal one for $\mu = 0$ but confines with $\mu \neq 0$. $\tilde{n}_c \equiv n_f - n_c$.

Deg.Freed.	Eff. Gauge Group	Phase	Global Symmetry
rel. nonloc.	-	Almost SCFT	$U(n_f)$
dual quarks	$USp(2\tilde{n}_c)\times U(1)^{n_c-\tilde{n}_c}$	Free Magnetic	$SO(2n_f)$

Table 2: Phases of $USp(2n_c)$ gauge theory with n_f flavors with $m_i \to 0$. $\tilde{n}_c \equiv n_f - n_c - 2$.

The 't Hooft - Mandelstam picture of confinement, caused by the condensation of monopoles in the maximal Abelian subgroup $U(1)^k$, ($k =$ Rank of the gauge group), is in fact realized only in some of the vacua. In a more "typical" vacuum of $SU(n_c)$ gauge theory, the effective, infrared degrees of freedom involve are a set of dual quarks, interacting with low-energy effective non-Abelian $SU(r)$ gauge fields. The condensation of these magnetic quarks as well as of certain Abelian monopoles also present in the theory, upon μ perturbation, lead to confinement and dynamical symmetry breaking. The semi-classical monopoles may be interpreted as baryonic composites made of these magnetic quarks and monopoles, which break up into their constituents before they become massless, as we move from the semiclassical region of the space of $N = 2$ vacua (parametrized by a set of gauge invariant VEVS) towards the relevant singularity.

The second most interesting result is that the special vacua in $SU(n_c)$ theory as well as the entire first group of vacua in $USp(2n_c)$ or $SO(n_c)$ theory correspond to various nontrivial infrared fixed

points (SCFT). The low-energy effective degrees of freedom in general contain relatively nonlocal states and there is no local effective Lagrangian description of these theories, though the symmetry breaking pattern can be found from the analysis at large adjoint mass μ.

Finally, in both type of gauge theories, for large number of flavors, there is a second group of vacua in free-magnetic phase, with no confinement and no spontaneous flavor symmetry breaking. In these vacua the low energy degrees of freedom are weakly interacting non-Abelian dual quarks and gauge particles, as well as some monopoles of products of $U(1)$ groups. In $SO(n_c)$ theories, the situation is qualitatively similar [8] to $USp(2n_c)$ cases; however, the effective gauge group and the unbroken global group in the vacua in free-magnetic phase are given by $SO(\tilde{n}_c) = SO(2n_f - n_c + 4)$ and $USp(2n_f)$, respectively, in these theories.

2 Analyses

Our analyses leading to these results consist of several independent steps [7, 8]:

i) Semi-classical analysis, yielding the number of the vacua, $\mathcal{N}$;

ii) Determination of dynamical symmetry breaking pattern at $\mu \gg \Lambda$ fixed and $m_i \to 0$;

iii) Check of the correct decoupling of the adjoint fields in the limit, $\mu \to \infty$, with m_i and $N = 1$ scale factor $\Lambda_1 \equiv \mu^{\frac{n_c}{3n_c - n_f}} \Lambda^{\frac{2n_c - n_f}{3n_c - n_f}}$ (for $SU(n_c)$ for instance) fixed;

iv) Study of $N = 1$ vacua at $m_i, \mu \ll \Lambda$, from the Seiberg-Witten curves, through the mass perturbation around CFT singularities [5];

v) Study of $N = 1$ vacua at $m_i, \mu \ll \Lambda$, by using the low-energy effective action [10, 11], leading to a clear microscopic picture of infrared physics;

vi) Numerical study of the maximal singularities of the Seiberg-Witten curves in the cases of rank 2 gauge groups ($SU(3)$, $USp(4)$, $SO(4)$, $SO(5)$);

vii) Study of monodromy around the singularities of the curves;

viii) Semi-classical analysis of the monopole flavor multiplet structure à la Jackiw-Rebbi.

These independent analyses lead, in a remarkably subtle way, to mutually consistent answers as regards the number of the vacua and physical properties of each of them. We shall discuss the two among the most important aspects of our analysis, ii) and v), below, but let us mention here the results of the semiclassical study, i). As the models have no flat directions, one can simply minimize the classical potential and get, after taking into account the appropriate Witten's index in case part of the gauge group remains unbroken by classical VEVS, the number of vacua. For $SU(n_c)$ theory with n_f flavors, there are

$$\mathcal{N} = \sum_{r=0}^{\min\{n_f, n_c - 1\}} (n_c - r) \binom{n_f}{r} \tag{2.1}$$

classical solutions. Note that when n_f is equal to or less than n_c the sum over r is done readily, and Eq. (2.1) is equal to

$$\mathcal{N}_1 = (2\,n_c - n_f)\, 2^{n_f - 1}, \qquad (n_f \le n_c)\,. \tag{2.2}$$

Similarly, we find for $USp(2n_c)$

$$\mathcal{N} = \sum_{r=0}^{min\{n_c, n_f\}} (n_c - r + 1) \cdot \begin{pmatrix} n_f \\ r \end{pmatrix} . \tag{2.3}$$

vacua, which reduces for smaller values of n_f to a closed expression,

$$\mathcal{N} = (2\, n_c + 2 - n_f)\, 2^{n_f - 1}, \qquad (n_f \leq n_c). \tag{2.4}$$

For $SO(n_c)$ theories the result is [8]

$$\mathcal{N} = \sum_{r=0}^{min\{[n_c/2], n_f\}} w(n_c - 2r) \begin{pmatrix} n_f \\ r \end{pmatrix} + \begin{pmatrix} n_f \\ n_c/2 \end{pmatrix}, \tag{2.5}$$

where

$$w(N) = N - 2, \quad N \geq 5, \tag{2.6}$$

and $w(N) = 4,\ 2,\ 1,\ 1,\ 1,$ for $N = 4,\ 3,\ 2,\ 1,\ 0,$ respectively, and the last term is present only for $2n_f \geq n_c,\ \ n_c =$ even. The formulas Eq.(2.1)-Eq.(2.4) correctly reduce to the well-known result

$$\mathcal{N} = n_f + 2, \tag{2.7}$$

in the case of the $SU(2)$ theory. Note that the generalization is nontrivial: e.g., $\mathcal{N} \neq n_f + n_c$ in $SU(n_c)$ theory! The complexity of the formulae Eq.(2.1) - Eq.(2.5) as compared to Eq.(2.7) signals indeed the presence of a rich variety of dynamical possibilities in general $SU(n_c)$, $USp(2n_c)$, and $SO(n_c)$ theories, some of which might well be important in the understanding of the standard QCD.

3 Determination of dynamical symmetry breaking pattern at large μ

At large μ ($\mu \gg \Lambda$), the effective superpotential can be read off from the bare Lagrangian by integrating out the heavy, adjoint fields and by adding to it the known exact instanton–induced superpotentials of the corresponding $N = 1$ theories. $N = 1$ supersymmetry guarantees that physics depend on μ holomorphically, so there cannot be any phase transition as μ is varied to smaller values.

3.1 $SU(n_c)$: $n_f \leq n_c + 1$

When the number of flavors is relatively small, the effective superpotential takes the form: [1]

$$W = -\frac{1}{2\mu} \left[\mathrm{Tr} M^2 - \frac{1}{n_c} (\mathrm{Tr} M)^2 \right] + \mathrm{Tr}(Mm) + (n_c - n_f) \frac{\Lambda_1^{(3n_c - n_f)/(n_c - n_f)}}{(\det M)^{1/(n_c - n_f)}}, \tag{3.1}$$

where $M^i_j \equiv \tilde{Q}_j Q^i$ are the $n_f \times n_f$ meson matrix, and

$$\Lambda_1 \equiv \mu^{\frac{n_c}{3n_c - n_f}} \Lambda^{\frac{2n_c - n_f}{3n_c - n_f}} \tag{3.2}$$

[1]To be precise, this is the form of the superpotential for generic $n_f < n_c$. For special cases $n_f = n_c$ and $n_f = n_c + 1$, one must use appropriate superpotentials involving baryonlike composites as well as mesons. See [7].

is the invariant mass scale of the $N=1$ SQCD (without the adjoint fields). The last term of Eq.(3.1) is the Affleck-Dine-Seiberg instanton-induced superpotential, the first arises from integrating out the adjoint fields Φ, $m = \mathrm{diag}(m_1, m_2, \ldots, m_f)$ is the bare quark mass matrix. After making an Ansatz, $\langle M \rangle = \mathrm{diag}(\lambda_1, \lambda_2, \ldots, \lambda_{n_f})$, one can straightforwardly find the minima of the potential. We find $(2n_c - n_f)_{n_f}C_r$ vacua in which the global symmetry is spontaneously broken (in $m_i \to 0$ limit) as

$$U(n_f) \to U(r) \times U(n_f - r). \tag{3.3}$$

By summing over r, $r = 0, 1, 2, \ldots [\frac{n_f}{2}]$, one finds $\mathcal{N}_1$ of Eq.(2.2).

3.2 $SU(n_c)$: $n_f \geq n_c + 2$

When the number of flavor exceeds $n_c + 1$, physics at low energies is described by the effective superpotential,

$$\mathcal{W} = \tilde{q} M q + \mathrm{Tr}(mM) - \frac{1}{2\mu}\left[\mathrm{Tr} M^2 - \frac{1}{n_c}(\mathrm{Tr} M)^2\right], \tag{3.4}$$

where q stands for the n_f dual quarks in the fundamental representation of the dual gauge group $SU(\tilde{n}_c)$, M is the meson matrix as in Eq.(3.1). By first minimizing the potential with respect to q and M, one finds

$$\mathcal{N}_2 = \sum_{r=0}^{\tilde{n}_c - 1} {}_{n_f}C_r \, (\tilde{n}_c - r) \tag{3.5}$$

of vacua, in which VEVS behave as

$$\langle q \rangle \to 0, \qquad \langle M \rangle \to 0, \tag{3.6}$$

in the limit, $m_i \to 0$. In other words, in these vacua the global $SU(n_f) \times U(1)$ symmetry remains unbroken.

One seems to encounter a puzzle though: the number of the vacua found here $\mathcal{N}_2$ is always less than the known total number of vacua, $\mathcal{N}$ (Eq.(2.1)). Where are other vacua?

Actually, we have tacitly assumed $\mathrm{Rank}\, M < n_f$ above, for otherwise the dual quarks are all massive and the theory reduces to the pure $SU(\tilde{n}_c)$ Yang-Mills in the infrared: its strong interaction dynamics must be taken into account in order to get information about its ground state.[2] In order to retrieve these vacua, we must first integrate out the dual quark fields. The instanton effects in the dual gauge group $SU(\tilde{n}_c)$ leads to a superpotential, which is identical to (actually continuation of) W in Eq.(3.1)! The minimization of such a potential yields $\mathcal{N}_1 = (2\, n_c - n_f)\, 2^{n_f - 1}$ vacua as before.

But then, for consistency, the sum of $\mathcal{N}_1$ and $\mathcal{N}_2$ must be equal to $\mathcal{N}$ of Eq.(2.1). As one can show easily by changing the dummy variable and by using the known identities among the binomial coefficients, this is indeed so.

[2] In fact, a related puzzle is how Seiberg's dual Lagrangian [6] - the first two terms of Eq. (3.4) - can give the right number of vacua for the massive $N=1$ SQCD with $n_f > n_c + 1$. By following the same method as below but with $\mu = \infty$, we do find the correct number (n_c) of vacua.

To sum up, in $SU(n_c)$ theories the exact global $U(n_f)$ symmetry in the equal mass (or massless) limit, which is spontaneously broken to $U(r)\times U(n_f-r)$ in $(2n_c-n_f)\,{}_{n_f}C_r$ vacua, $r=0,1,\ldots,[n_f/2]$. When the number of the flavor is larger ($n_f > n_c+1$), we find another class of vacua, with no global symmetry breaking. We shall see that these match the vacua in the free-magnetic phase at small μ.

3.3 $USp(2n_c)$, $SO(n_c)$

The analysis in the cases of $USp(2n_c)$ or $SO(n_c)$ theories is similar, although the results are qualitatively different from the case of $SU(n_c)$ theory.

In $USp(2n_c)$ (or $SO(n_c)$) theories, for small numbers of flavors, *the chiral $SO(2n_f)$ (or $USp(2n_f)$) symmetry in the massless limit is always spontaneously broken to $U(n_f)$.* This result nicely agrees with what is expected generally from bi-fermion condensate of the standard form in non supersymmetric theories, and forms a result in closest analogy with what is supposed to occur in the standard QCD with small number of flavors.

Finally, in the cases of $USp(2n_c)$ or $SO(n_c)$ theories too, there exist also other vacua without dynamical symmetry breaking, when the number of flavor is greater ($n_f > n_c+2$ or $2n_f > n_c-4$, respectively).

4 Quantum vacua at $\mu \ll \Lambda$

At small μ, the infrared properties of the theory are described by certain singularities of Seiberg-Witten curves [1, 2, 4]. To be concrete take the case of $SU(n_c)$ gauge theory with n_f flavors. $N=1$ supersymmetric vacua are found by requiring that the curve

$$y^2 = \prod_{k=1}^{n_c}(x-\phi_k)^2 + 4\Lambda^{2n_c-n_f}\prod_{j=1}^{n_f}(x+m_j), \tag{4.1}$$

where

$$\langle\phi\rangle = \mathrm{diag}(\phi_1,\phi_2,\ldots), \tag{4.2}$$

describe the gauge invariant VEVS,

$$u = \langle \mathrm{Tr}\Phi^2\rangle = \langle \sum_{i<j}\phi_i\phi_j\rangle, \quad u_3 = \langle \mathrm{Tr}\Phi^3\rangle = \langle \sum_{i<j<k}\phi_i\phi_j\phi_k\rangle, \tag{4.3}$$

etc., to be maximally singular. Since there are n_c-1 free parameters, up to n_c-1 pairs of branch points can be made to coincide by appropriate choices of $\{\phi\}$, and this corresponds to the condition that there are maximal number of massless monopoles of Abelian subgroup $U(1)^{n_c-1}\subset SU(n_c)$.

Such a connection follows from the by now well understood association of monopole masses with the integral over canonical cycles of meromorphic differentials on the curve such as Eq.(4.1) [1, 2, 4]. Also one can show that, upon μ perturbation, only these singular points of QMS (quantum space of vacua) lead to supersymmetric ground states.

Once the low-energy degrees of freedoms (monopoles) are identified and their quantum numbers known, it is in principle straightforward to analyze the properties of the vacua.

It turns out that the limit $m_i \to 0$ is highly nontrivial, and physics in the infrared is far richer than one might have expected from the knowledge of $SU(2)$ gauge theory [2]. In particular, one finds that the low-energy degrees of freedom are not always the monopoles of the maximal Abelian subgroup envisaged in the Nambu-'t Hooft-Mandelstam mechanism.

The crucial steps of our analysis are the points iv) and v) of Sec. 2. The first of these steps shows how all the $N = 1$ vacua, selected out by the adjoint mass perturbation, are associated with the various universality classes of SCFT [5], and allows us to relate the quantum vacua at small μ to those at large μ; the second step leads to the microscopic picture of confinement and dynamical symmetry breaking, summarized below.

5 Microscopic Picture of Dynamical Symmetry Breaking

In $SU(n_c)$ theories with n_f flavors, there are two group of vacua. In the first group of vacua with finite meson or dual quark vacuum expectation values (VEVS), labeled by an integer r, $r \leq [n_f/2]$, the system is in confinement phase. The nature of the actual carrier of the flavor quantum numbers depends on r. In vacua with $r = 0$, magnetic monopoles are singlets of the global $U(n_f)$ group, hence no global symmetry breaking accompanies confinement.

In vacua with $r = 1$, the light particles are magnetic monopoles in the fundamental representation of $U(n_f)$ flavor group. Their condensation leads to the confinement and flavor symmetry breaking, simultaneously.

In vacua labeled by r, $2 \leq r < n_f/2$ $(r \neq n_f - n_c)$, the grouping of the associated singularities on the Coulomb branch, with multiplicity, ${}_{n_f}C_r$, at first sight suggests the condensation of monopoles in the rank-r anti-symmetric tensor representation of the global $SU(n_f)$ group. Actually, this does not occur. The low-energy degrees of freedom of these theories are n_f magnetic quarks (in $\underline{r}$) plus a number of singlet monopoles of a non-Abelian effective $SU(r) \times U(1)^{n_c - r}$ gauge theory [10].

Monopoles in higher representations of $SU(n_f)$ flavor group, even if they exist semi-classically, break up into magnetic quarks before they become massless at singularities on the Coulomb branch. It is the condensation of the latter that induces confinement and flavor symmetry breaking, $U(n_f) \to U(r) \times U(n_f - r)$, in these vacua. The system thus realizes the global symmetry of the theory in a Nambu-Goldstone mode, without having unusually many Nambu-Goldstone bosons. It is a novel mechanism for confinement and dynamical symmetry breaking.

In the special cases with $r = n_f/2$, still another dynamical scenario takes place. In these cases, the interactions among the monopoles and dyons become so strong that the low-energy theory describing them is a nontrivial SCFT, with conformal invariance explicitly broken by the adjoint or quark masses. Although the symmetry breaking pattern is known $U(n_f) \to U(n_f/2) \times U(n_f/2)$, the low-energy degrees of freedom in general involve relatively nonlocal fields and their interactions cannot be described in terms of a local action.

Finally, there are vacua in which magnetic-quarks do not condense and remain as physically observable particles at long distances, interacting with non-Abelian dual gauge particles (free magnetic phase), when the number of the flavor n_f exceeds $n_c + 1$. The global $U(n_f)$ symmetry remains

unbroken in these vacua. These precisely match the second group of vacua found at large μ in which no condensate forms.

In $USp(2n_c)$ theories, again, we find two groups of vacua, whose properties are shown in Table 2. The most salient difference as compared to the $SU(n_c)$ theory is that here the entire first group of vacua corresponds to a SCFT. It is a nontrivial superconformal theory: one does not have a local effective Lagrangian description for those theories.[3] Nonetheless, the symmetry breaking pattern can be deduced, from the analysis done at large μ: $SO(2n_f)$ symmetry is always spontaneously to $U(n_f)$.

To see better what is going on, it is instructive to consider the equal but nonvanishing quark mass case first. See Table 3. The flavor symmetry group of the underlying theory is now explicitly broken to $U(n_f)$. The first group of vacua split into various subgroups of vacua labeled by r, $r = 0, 1, 2, \ldots, [\frac{n_f-1}{2}]$, each of which is described by a *local* effective gauge theory of Argyres-Plesser-Seiberg [10] for $SU(n_c)$ theory (!), with gauge group $SU(r) \times U(1)^{n_c-r+1}$ and n_f (dual) quarks in the fundamental representation of $SU(r)$.[4] Indeed, the gauge invariant composite VEVS characterizing these vacua differ by some positive powers of m, so that the validity of each effective theory is limited to small fluctuations of order of $m^a, \ a > 0,$ around each vacuum.

In the limit $m \to 0$ these points in the quantum moduli space (QMS) collapse into one single point and accordingly the range of the validity of each local effective action shrinks to zero, implying that we have a nontrivial SCFT here, with mutually nonlocal massless states. In the example of $USp(4)$ theory with $n_f = 4$, we have explicitly verified this by determining the singularities and branch points at finite equal mass m and then by studying the limit $m \to 0$.

Deg.Freed.	Eff. Gauge Group	Phase	Global Symmetry
monopoles	$U(1)^{n_c}$	Confinement	$U(n_f)$
monopoles	$U(1)^{n_c}$	Confinement	$U(n_f-1) \times U(1)$
dual quarks	$SU(r) \times U(1)^{n_c-r+1}$	Confinement	$U(n_f-r) \times U(r)$
dual quarks	$SU(\frac{n_f}{2}) \times U(1)^{n_c-\frac{n_f}{2}+1}$	SCFT	$U(\frac{n_f}{2}) \times U(\frac{n_f}{2})$

Table 3: The first group of vacua of $USp(2n_c)$ theory with n_f flavors with $m_i = m \neq 0$. The vacuum label r in the third row runs for $r = 2, 3, \ldots, [\frac{n_f-1}{2}]$.

The first group of vacua in the $SO(n_c)$ gauge theory has similar characteristics as the one in $USp(2n_c)$ theory just discussed. These cases, together with the special $r = n_f/2$ vacua for the $SU(n_c)$ theory, reveal another new mechanism for dynamical symmetry breaking: although the global symmetry breaking pattern deduced indirectly looks familiar enough, the low-energy degrees of freedom are relatively nonlocal dual quarks and dyons. It would be interesting to get a better

[3]Except for $n_f = 3$ and $n_f = 2$, in which cases we expect a local description to be valid, see Sec. 8.2 of Carlino et. al. [7].

[4]The key fact that some of the SCFT in $USp(2n_c)$ or in $SO(n_c)$ theories at $m_i = m \neq 0$ are in the same universality classes as those occurring in the $SU(n_c)$ theory, was first noted by Eguchi et. al. [5]. Our perturbation theory in masses around the SCFT and the effective action analysis show that these SCFT are indeed the $N = 1$ vacua which survive the adjoint mass perturbation (missed in the analysis of [11]) .

understanding of this phenomenon.

For large numbers of flavor, there are also vacua, just as in large n_f $SU(n_c)$ theories, with no confinement and no dynamical flavor symmetry breaking. Physics around these vacua can be explicitly studied by use of the effective low-energy action at the "special points" of [11]. The low-energy particles are solitonlike magnetic quarks which weakly interact with dual (in general) non-Abelian gauge fields: the system is in the free magnetic phase. These are smoothly connected to the second group of vacua, with no symmetry breaking, found at large μ.

Properties of various vacua in $SU(n_c)$ and $USp(2n_c)$ theories are illustrated schematically in Fig. 1 and Fig. 2.

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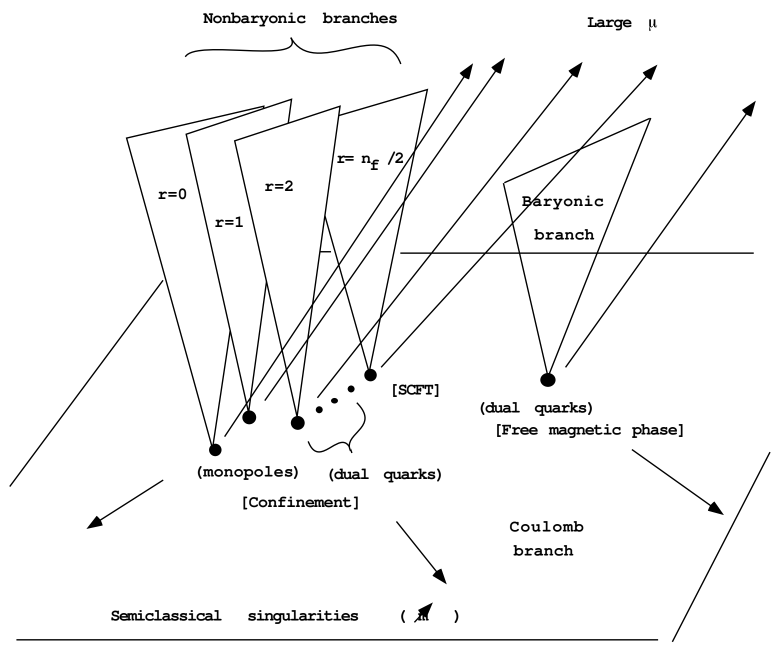

Figure 1: A schematic view of the $N = 2$ space of vacua (QMS) in $SU(n_c)$ theories and its singularities corresponding to $N = 1$ vacua (black dots). The latter lie near the roots of various Higgs branches. When the generic quark masses are added each of $N = 1$ vacuum further splits. At large quark masses these points move to semiclassical regions of Coulomb branch.

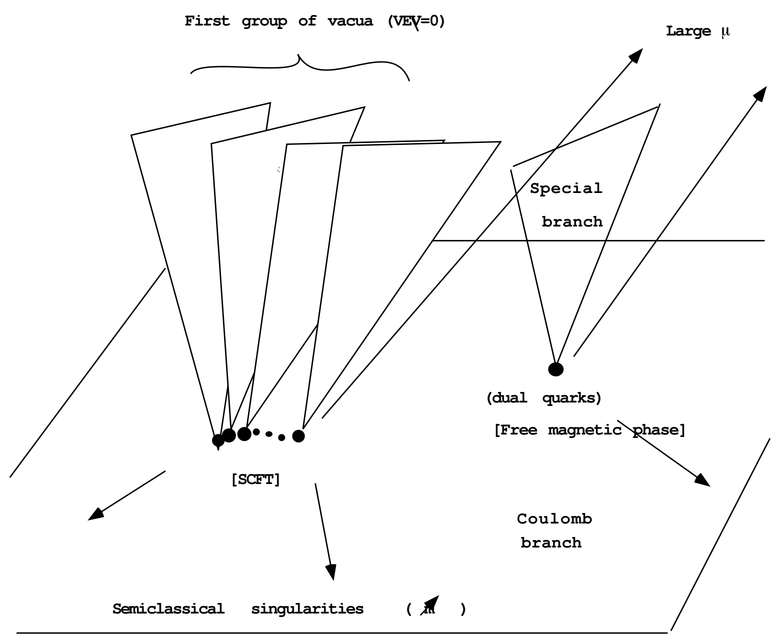

Figure 2: $N=2$ space of vacua (QMS) and the singularities corresponding to $N=1$ vacua (black dots) near the roots of various Higgs branches, in $USp(2n_c)$ theories. For $m_i = m \neq 0$ the situation is similar to $SU(n_c)$ case; in the $m \to 0$ limit, however, the various vacua of the first group collapse into one single vacuum, leading to a nontrivial fixed point behavior.